1.32) $B_1 := \left\{ \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 9 \\ 11 \end{bmatrix} \right\}$

a) Queremos buscar B_2 tal que:

$$M_{B_1B_2} = \begin{bmatrix} 5 & 5 & 10 \\ 0 & 5 & 5 \\ 0 & 0 & 9 \end{bmatrix}$$

Sabemos que, por teorema:

$M_{B_1B_2} = M_{CB_2} \, M_{B_1C}$ siendo C la base canónica de $\mathbb{R}^3$.

[redacted]

Como yo quiero calcular M_{B_2C}, puedo, con el teorema, calcular M_{CB_2} y luego hallar su inversa, ya que $(M_{CB_2})^{-1} = M_{B_2C}$.

Entonces:

$$\begin{bmatrix} 5 & 5 & 10 \\ 0 & 5 & 5 \\ 0 & 0 & 9 \end{bmatrix} = (\bar{u}, \bar{v}, \bar{w}) \cdot \begin{bmatrix} 3 & -1 & 2 \\ 0 & 0 & 9 \\ 4 & 7 & 11 \end{bmatrix}$$

Ecuaciones:

$$\begin{cases} 3\bar{u} + 4\bar{w} = (5\ 0\ 0) & \text{I} \\ -\bar{u} + 7\bar{w} = (5\ 5\ 0) & \text{II} \\ 2\bar{u} + 9\bar{v} + 11\bar{w} = (10\ 5\ 9) & \text{III} \end{cases}$$

(NO SÉ SI ARMÉ BIEN LAS ECUACIONES)

de I $\rightarrow \bar{u} = \dfrac{(5\ 0\ 0) - 4\bar{w}}{3}$

en II $\rightarrow \left(-\frac{5}{3}\ 0\ 0\right) + \frac{4}{3}\bar{w} + 7\bar{w} = (5\ 5\ 0) \rightarrow$

$\rightarrow \frac{25}{3}\bar{w} = (5\ 5\ 0) - \left(-\frac{5}{3}\ 0\ 0\right) \rightarrow \frac{25}{3}\bar{w} = \left(\frac{20}{3}\ 5\ 0\right) \rightarrow$

$\rightarrow \bar{w} = \frac{3}{25} \cdot \left(\frac{20}{3}\ 5\ 0\right) \rightarrow \left[\bar{w} = \left(\frac{4}{5}\ \frac{3}{5}\ 0\right)\right]$

em I $\rightarrow \bar{u} = \dfrac{(5\ 0\ 0) - 4\cdot\left(\frac{4}{5}\ \frac{3}{5}\ 0\right)}{3} \rightarrow \bar{u} = \left(5 - \frac{16}{5}, \frac{-12}{5}, 0\right)\cdot\frac{1}{3} \rightarrow$

$\rightarrow \bar{u} = \left(\frac{9}{5}, \frac{-12}{5}, 0\right)\cdot\frac{1}{3} \rightarrow \left[\bar{u} = \left(\frac{3}{5}, \frac{-4}{5}, 0\right)\right]$

em III $\rightarrow 2\cdot\left(\frac{3}{5}, \frac{-4}{5}, 0\right) + 9\bar{v} + 11\cdot\left(\frac{4}{5}, \frac{3}{5}, 0\right) = (10, 5, 9) \rightarrow$

$\rightarrow \left(\frac{6}{5}, \frac{-8}{5}, 0\right) + 9\bar{v} + \left(\frac{44}{5}, \frac{33}{5}, 0\right) = (10, 5, 9) \rightarrow$

$\rightarrow (10, 5, 0) + 9\bar{v} = (10, 5, 9) \rightarrow 9\bar{v} = (0, 0, 9) \rightarrow \left[\bar{v} = (0, 0, 1)\right]$

Armo M_{CB2}: $M_{CB2} = [\bar{u}\ \bar{v}\ \bar{w}] \rightarrow M_{CB2} = \begin{bmatrix} \frac{3}{5} & 0 & \frac{4}{5} \\ \frac{-4}{5} & 0 & \frac{3}{5} \\ 0 & 1 & 0 \end{bmatrix}$

Busco su inversa:

$$\left(\begin{array}{ccc|ccc} \frac{3}{5} & 0 & \frac{4}{5} & 1 & 0 & 0 \\ \frac{-4}{5} & 0 & \frac{3}{5} & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right) F2 \leftrightarrow F3$$

$$\left(\begin{array}{ccc|ccc} \frac{3}{5} & 0 & \frac{4}{5} & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ \frac{-4}{5} & 0 & \frac{3}{5} & 0 & 1 & 0 \end{array}\right) \frac{4}{5}F1 + \frac{3}{5}F3 \quad \left(\begin{array}{ccc|ccc} \frac{3}{5} & 0 & \frac{4}{5} & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 4/5 & 3/5 & 0 \end{array}\right) \frac{4}{5}F3 - F1$$

$$\left(\begin{array}{ccc|ccc} -\frac{3}{5} & 0 & 0 & \frac{-9}{25} & \frac{12}{25} & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & \frac{4}{5} & \frac{3}{5} & 0 \end{array}\right) F1/-3/5 \quad \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{5} & -\frac{4}{5} & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & \frac{4}{5} & \frac{3}{5} & 0 \end{array}\right)$$

$\boxed{\text{Por lo tanto } B_2 = \left\{ \begin{bmatrix} 3/5 \\ 0 \\ 4/5 \end{bmatrix}, \begin{bmatrix} -4/5 \\ 0 \\ 3/5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}}$

1.32) b) Quiero hallar M_{CBZ}.

Se que $(M_{BZC})^{-1} = M_{CBZ}$

y por el punto a) lo que calculé antes de hacer la inversa es $M_{CBZ} \rightarrow$ $\boxed{M_{CBZ} = \begin{bmatrix} \frac{3}{5} & 0 & \frac{4}{5} \\ -\frac{4}{5} & 0 & \frac{3}{5} \\ 0 & 1 & 0 \end{bmatrix}}$

Ahora quiero $[x_1 \; x_2 \; x_3]_{B_2}$, lo escribo como cl de B_2:

$$(x_1, x_2, x_3) = \alpha_1 . (3/5, 0, 4/5) + \alpha_2 . (-4/5, 0, 3/5) + \alpha_3 . (0, 1, 0)$$

Ec.:

$$\begin{cases} \frac{3}{5}\alpha_1 - \frac{4}{5}\alpha_2 = x_1 \rightarrow \alpha_1 = \left(x_1 + \frac{4}{5}\alpha_2\right) . \frac{5}{3} \rightarrow \alpha_1 = \frac{5}{3}x_1 + \frac{4}{3}\alpha_2 \quad \text{I} \\ \boxed{\alpha_3 = x_2} \quad \text{II} \\ \frac{4}{5}\alpha_1 + \frac{3}{5}\alpha_2 = x_3 \quad \text{III} \end{cases}$$

I em III $\rightarrow \frac{4}{5} . \left(\frac{5}{3}x_1 + \frac{4}{3}\alpha_2\right) + \frac{3}{5}\alpha_2 = x_3 \rightarrow \frac{4}{3}x_1 + \frac{16}{15}\alpha_2 + \frac{3}{5}\alpha_2 = x_3 \rightarrow$

$\rightarrow \frac{4}{3}x_1 + \frac{5}{3}\alpha_2 = x_3 \rightarrow \alpha_2 = \left(x_3 - \frac{4}{3}x_1\right) . \frac{3}{5} \rightarrow \boxed{\alpha_2 = \frac{3}{5}x_3 - \frac{4}{5}x_1}$

em I $\rightarrow \alpha_1 = \frac{5}{3}x_1 + \frac{4}{3} . \left(\frac{3}{5}x_3 - \frac{4}{5}x_1\right) \rightarrow \alpha_1 = \frac{5}{3}x_1 + \frac{4}{5}x_3 - \frac{16}{15}x_1 \rightarrow$

$\rightarrow \boxed{\alpha_1 = \frac{3}{5}x_1 + \frac{4}{5}x_3}$

Por lo tanto: $\boxed{[x_1 \; x_2 \; x_3]_{B_2} = \begin{bmatrix} \frac{3}{5}x_1 + \frac{4}{5}x_3 \\ \frac{3}{5}x_3 - \frac{4}{5}x_1 \\ x_2 \end{bmatrix}}$